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DETERMINATION OF THE EQUATION OF STATE
OF EXPLOSIVE DETONATION PRODUCTS FROM
THE CYLINDER EXPANSION TEST

John F. Polk

July 1984



US ARMY ARMAMENT RESEARCH AND DEVELOPMENT CENTER
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I. INTRODUCTION

The cylinder expansion test,¹ in which the explosively-driven, outward radial expansion of a standard metal cylinder is observed by a streak camera, has become one of the classic experimental tools in research concerned with detonation dynamics. In fact a large portion of our practical and reliable information about explosives has been obtained directly or indirectly from this experiment; Gurney velocities, Taylor angles and detonation velocities can all be deduced from cylinder expansion test data. In practice, however, the primary use of such data has been in determining equations of state (EOS) for the detonation product gases, expressed as a curve in the pressure-density plane describing the Chapman-Jouguet isentrope. This is normally accomplished by an iterative procedure in which a full-scale continuum mechanics computer code, such as HEMP, is used to model the observed experiment. The equation of state in the codes is written in an assumed analytic expression involving a number of undetermined parameters which can be adjusted repeatedly with successive computer calculations until "good" agreement with the experimental measurements is obtained. The resulting parameter values are then considered as correct for the particular explosive and can reasonably be used in subsequent calculations. Viewed in this light, the computer codes can be seen as very elaborate fitting and extrapolation procedures.

Among the better known expressions for the equations of state of gases are the polytropic gas law (constant γ), LJD (Lennard-Jones-Devonshire), BKW (Becker-Kistiakowsky-Wilson), and JWL (Jones-Wilkins-Lee). Most of the presently available codes will allow the user to select the form which he considers most appropriate and also contain a compilation of appropriate parameter values for commonly used explosives. (See References 2 and 3.)

¹J. W. Kury et al., "Metal Acceleration by Chemical Explosives," *Proceedings of the Forth Symposium on Detonation*, Office of Naval Research Report ACR-126, pp3-12 (1965).

²Dobratz, Brigitta M., "Properties of Chemical Explosives and Explosive Stimulants," UCRL 51319, Dec 1972, Lawrence Livermore Laboratory, Livermore CA.

³Lee, E.L., Hornig, H.C., Kury, J.W., "Adiabatic Expansion of High Explosive Detonation Products," UCRL 50422, May 1968, Lawrence Livermore Laboratory, Livermore, CA.

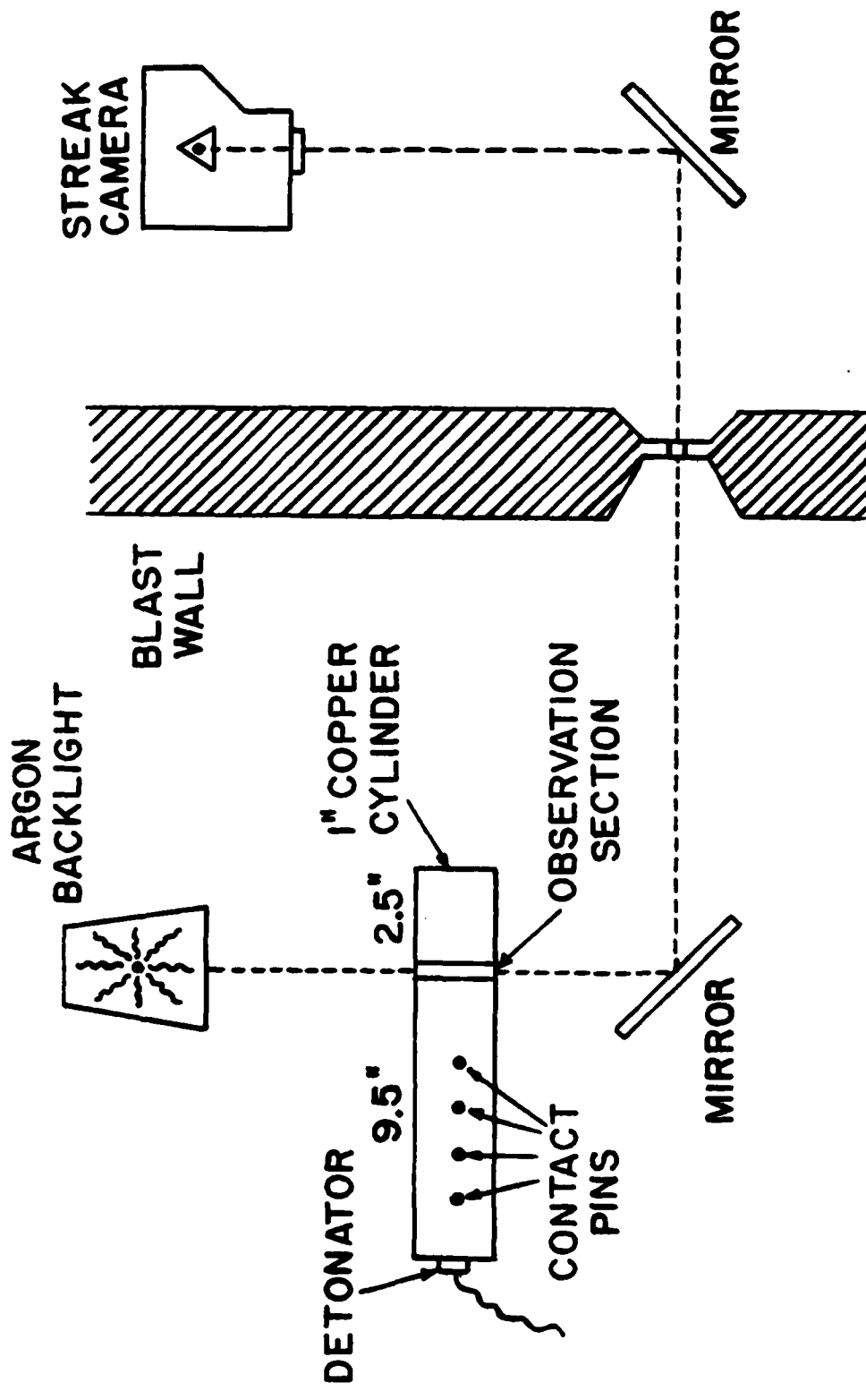


Figure 1. Schematic of Cylinder Expansion Test

In the present study we propose an alternative approach for deducing equation of state data directly from the cylinder expansion test results. This procedure is basically a reversed version of the well-known analysis of cylindrical bombs due to G. I. Taylor.⁴ It does not require the use of a large scale continuum mechanics code, nor does it assume an analytic expression for the equation of state with adjustable parameters, but computes pressure directly in tabular form as a function of density. To implement this procedure a computer program called "CEDAR" (Cylinder Expansion Data Reduction) was written in the BASIC programming language for the Hewlett Packard Model 9845B minicomputer; execution time for this program is typically less than one minute. A tabulation of CEDAR is contained in Appendix A and an example of an output is shown in Appendix B.

II. EXPERIMENTAL SET UP

To provide a general framework for discussion, let us first describe the experimental set up and procedure for the cylinder expansion test as it is conducted in the Warhead Mechanics Branch of the Ballistics Research Laboratory. A schematic is shown in Figure 1.

A long, hollow, open-ended cylinder is packed with explosive to a desired loading density and a detonator and booster charge placed at one end. The most common choice for this test is a 12 inch long copper cylinder, with a one inch interior diameter and 0.1 inch wall thickness. Interior diameters of two and four inches are also considered standard and results from one geometry can be made comparable with others through appropriate similarity variables, as in Reference 1. The program listed in Appendix A can be used for any choice of inner and outer cylinder diameters. In the tests performed at Lawrence Livermore National Laboratories great care was taken to fabricate the cylinders with exact dimensions. At BRL, much more economical stock

⁴G.I. Taylor, "Analysis of the Explosion of a Long Cylindrical Bomb Detonated at one End," *The Scientific Papers of Sir G. I. Taylor, Volume III*, p277; edited by G.K. Batchelor, Cambridge University Press, 1963.

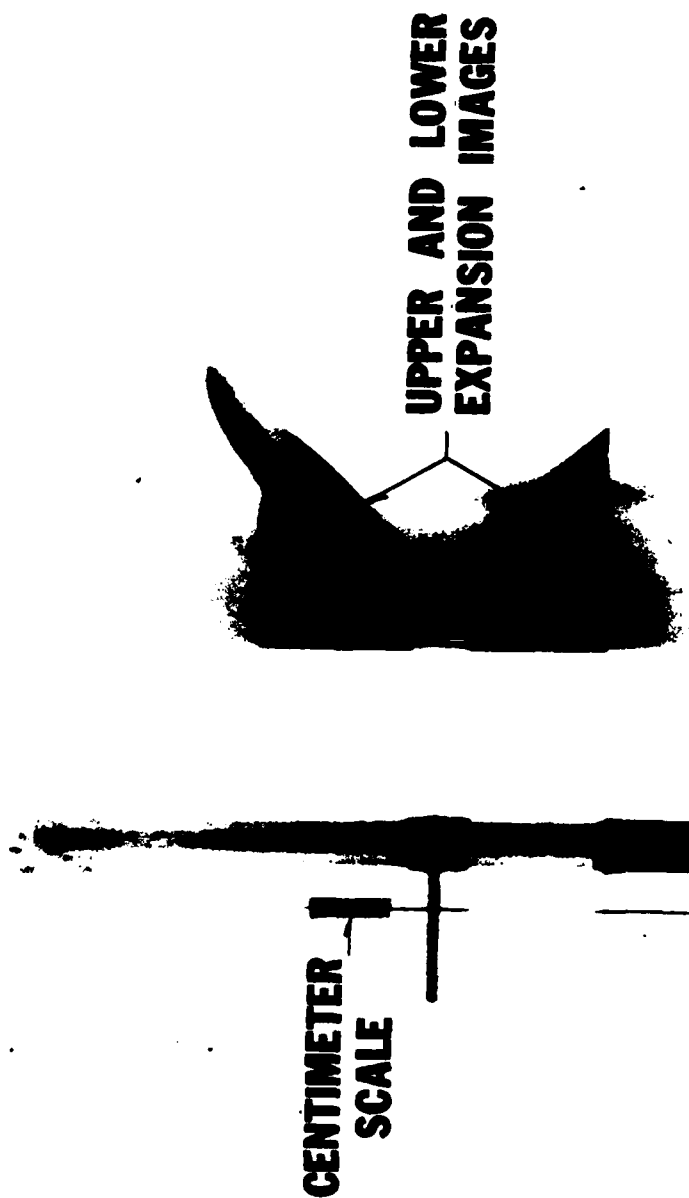


Figure 2. Streak Camera Film Negative

copper pipe was purchased from local sources and machined to proper dimensions, practically identical results were obtained.

A Beckman and Whitley Model 168 streak camera is placed behind a blast wall and situated to observe, using a mirror, the transverse expansion of a chosen cross section of the cylinder; this section is located so that a fully developed steady-state detonation front, with no end effects, prevails during the period of observation. For the standard 12 in. cylinder, the observed station was positioned 9.5 in. from the initiated end.

As the detonation wave passes through the observed section, the cylinder walls are driven outwards and obscure the backlighting provided by an argon light bomb. The external diameter at that section is continuously observed by the camera through a slit aperture and its image swept at uniform velocity across the recording film by means of a rotating drum mirror. A typical film negative resulting from this procedure is shown in Figure 2. The vertical scale is proportional to the cylinder wall displacement. (Both top and bottom walls are observed and have virtually identical behavior.) The proper scale factor for converting film measurements to actual displacements is determined from the millimeter scale, appearing at the left, which was photographed before initiation. Measurements along the horizontal axis correspond to elapsed time; the appropriate scale factor can be determined from knowledge of the camera dimensions and mirror rotation rate.

When a film record, such as that in Figure 2, becomes available from an experiment, it is placed in a film reader and x and y coordinates are read from both top and bottom displacement curves. These are compared for discrepancies, averaged and multiplied by scale factors to yield a curve $d(t)$ which expresses the exterior cylinder wall displacement as a function of time.

As an additional feature, the detonation velocity of the explosive can be determined during the experiment by placing several electrical contact pins or break wires at measured distances along the cylinder. At our facility four contact pins were positioned at 2 in. intervals beginning at 2.5 in. from the initiating end. The elapsed time between the making or breaking of contact

at different stations is recorded as voltage on an analog device during the experiment and quantified afterwards using an oscilloscope. Some care must be taken to ensure that the explosive is packed at uniform density along the entire length of the pipe. In our case this was accomplished by forming the explosive separately in 2 in. sections with the proper loading density. These were then placed firmly together in the cylinder, but without additional compaction.

III. ANALYSIS

Determination of an equation of state for the detonation product gases requires an understanding of the explosion gas dynamics, its interaction with the cylinder walls and the material strength of the cylinder at high strain rates. The discussion by G. I. Taylor⁴ provides a useful and workable model of this process and will serve as a basis for the present analysis. In addition, we have developed a methodology for extracting the cylinder wall motion in more detail than is directly available from the experimental record. Together these enable us to attain our basic objective -- determination of pressure as a function of density (or specific volume) along the Chapman-Jouguet isentrope. Our approach involves three subtasks which will be discussed separately in the following sections: kinematics of the cylinder wall motion, parametric determination of the EOS and considerations relating to numerical analysis.

A. Kinematics of the Cylinder Wall Motion

The curve $d = d(t)$ obtained from the experimental film readings is a record of the apparent cylinder wall displacement and not a direct measurement of the motion of an individual wall particle. This measured displacement can be considered as Eulerian whereas the path of an actual wall particle is more properly described by a Lagrangian coordinate system in which both the axial and radial displacements are expressed as functions of time. In order to compare wall particle trajectories directly with code predictions and to compute accelerations more accurately, it is necessary to deduce this Lagrangian information from knowledge of $d(t)$ only. In the following discussion we shall

show how this can be done.

To begin, we define our terms more precisely. Let z and r denote axial and radial coordinates in an Eulerian system chosen so that $z = 0$ at the observation station and $r = 0$ along the central axis of the cylinder. We suppose that the inner and exterior radii of the test cylinder at station $z = 0$ are given by $R_a(t)$ and $R_e(t)$ which are initially (at time $t \leq 0$) equal to R_0 and R_1 respectively. The experimental data measures the displacement of the outer surface from rest; thus,

$$R_a(t) = R_1 + d(t) . \quad (1)$$

To determine $R_a(t)$ we assume incompressibility of the cylinder wall material during acceleration. This implies

$$\pi(R_e^2 - R_a^2) = A_0 \equiv \pi(R_1^2 - R_0^2)$$

or

$$R_a(t) = \left[R_e^2(t) - A_0/\pi \right]^{1/2} . \quad (2)$$

The acceleration of the wall should be calculated not at the inner or outer surface but at the central surface defined by

$$R_m(t) \equiv \left[R_e^2(t) - A_0/2\pi \right]^{1/2} .$$

Let us next assume that the expansion is steady state, that is it propagates with a fixed waveform along the cylinder at a constant velocity D , the explosive detonation velocity. Thus the motion at any axial station z is related to that at $z = 0$ through the similarity variable $(t - z/D)$; in particular, the apparent central surface displacement from its initial position at station z observed at time t is

$$h(z,t) \equiv \begin{cases} 0 & t \leq z/D \\ R_m(t-z/D) - R_m(0) & t \geq z/D. \end{cases} \quad (3)$$

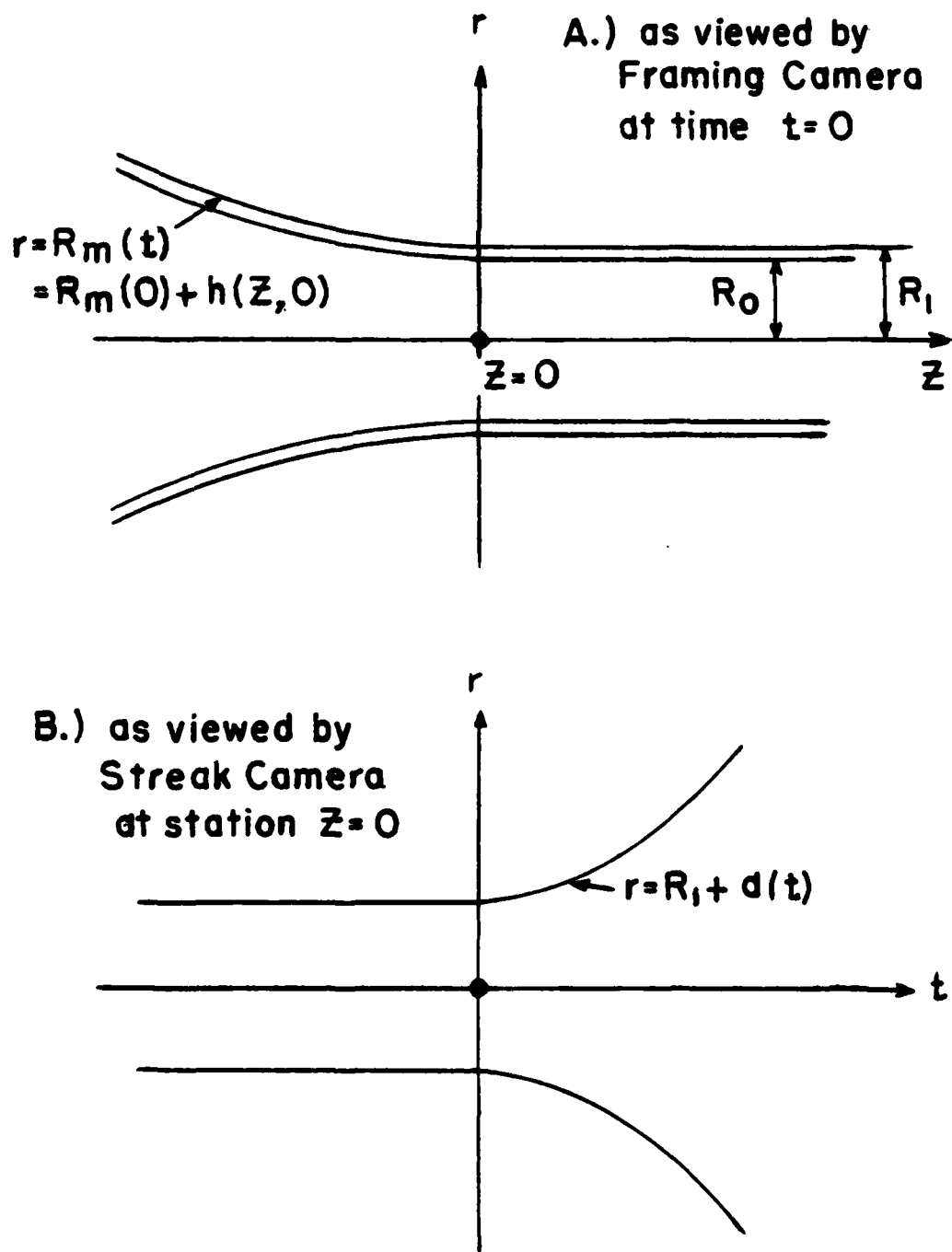


Figure 3. Steady State Cylinder Expansion as Observed from
a) Framing Camera and b) Streak Camera

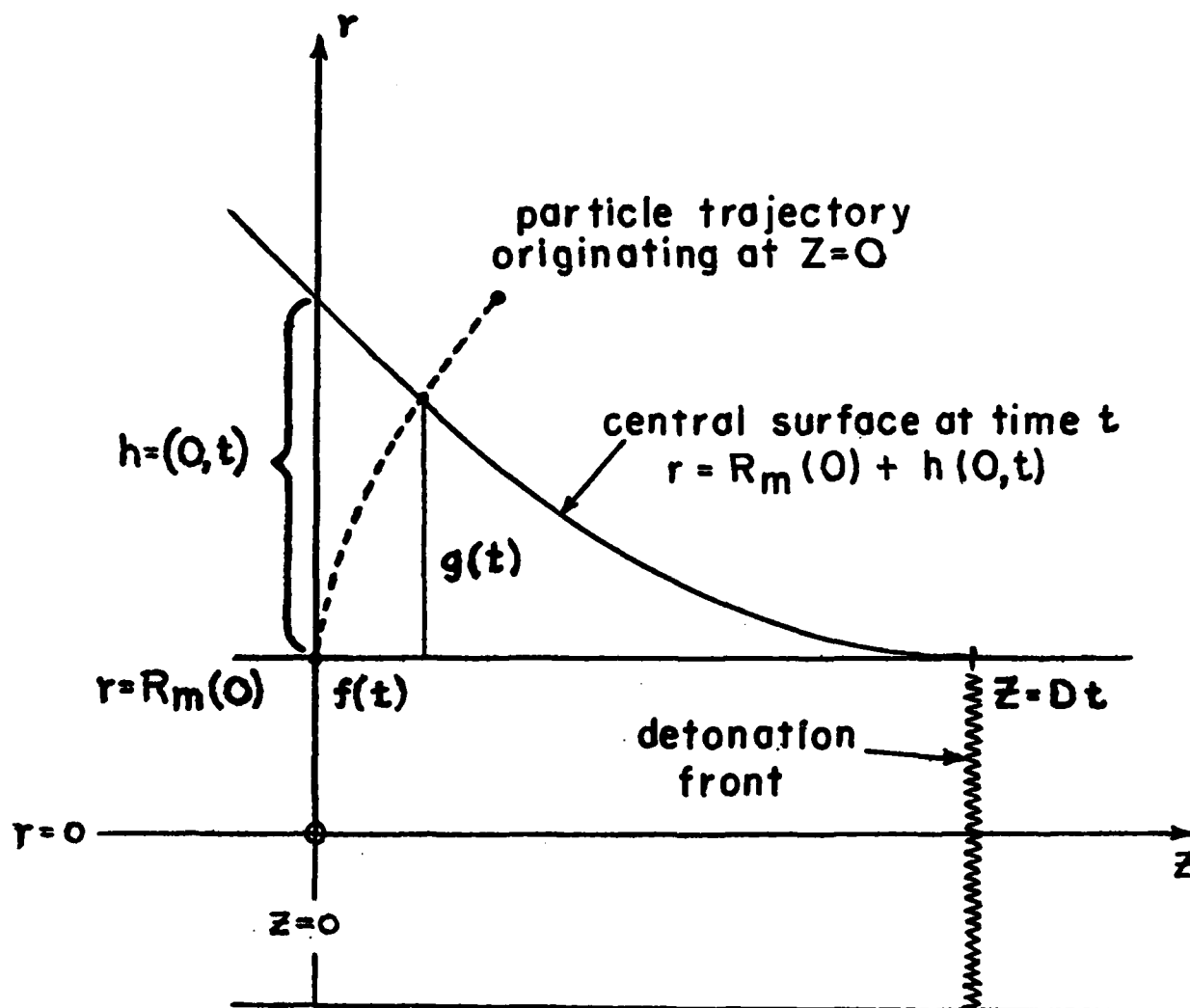
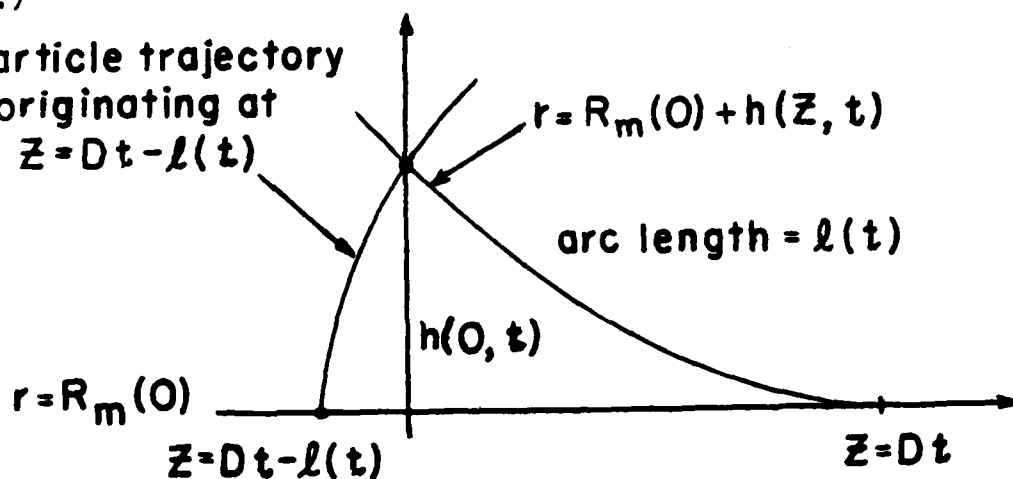


Figure 4. Kinematics of Wall Particle Motion, Viewed in Eulerian Framework.

A.)

particle trajectory
originating at
 $z = Dt - l(t)$



B.)

particle
trajectory
originating
at $z = 0$

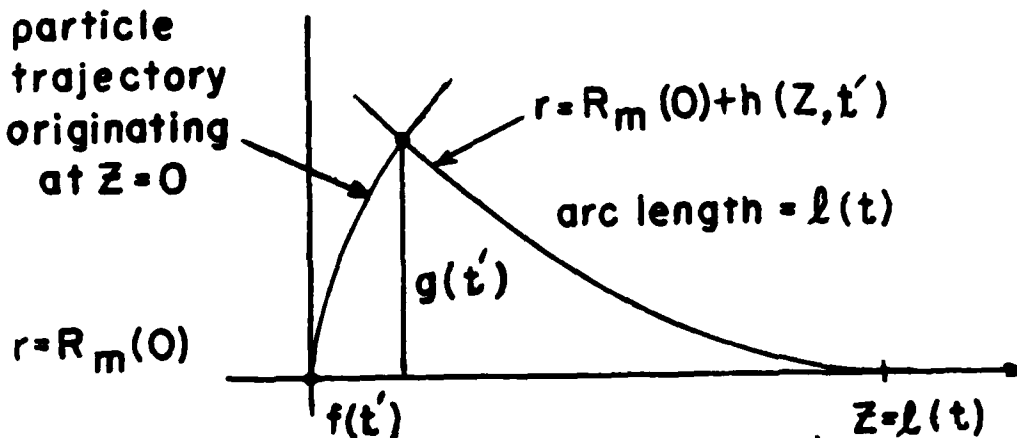


Figure 5. Comparison of Two Different Particle Trajectories
a) particle observed at $z = 0$ at time t b) particle
originating at $z = 0$, observed at time $t' = l(t)/D$

Considered as a function of z only, this describes the central wall displacement as it would appear to a framing camera activated at time t ; as a function of t , it gives the displacement that would be viewed by a streak camera located at station z . These two different frames of reference are shown schematically in Figure 3.

The wall particle initially located at station $z = 0$ is driven outwards along a path which might be described parametrically in the form

$$\begin{aligned} z &= f(t) \\ r &= g(t) + R_m(0), \end{aligned} \quad (4)$$

as indicated in Figure 4. The steady state assumption establishes a relationship between these functions when $d(t)$ is known since

$$g(t) = h(f(t), t) = R_m(t - f(t)/D) - R_m(0). \quad (5)$$

However, this is insufficient to determine f and g uniquely unless an additional assumption is made. Accordingly, we suppose, as in Taylor's⁴ analysis, that pressure is exerted normally and consequently that no longitudinal deformation of the cylinder walls occurs. Mathematically this means that the distance between wall particles measured as arc-length along the central surface remains constant during the expansion. We therefore introduce the function

$$l(t) = \int_0^{Dt} \left[1 + \left(\frac{\partial h}{\partial z} \right)^2 \right]^{1/2} dz$$

which, referring to Figure 5.a, is seen as the arc-length observed at time t between the detonation front, at $z = Dt$, and the observation station $z = 0$. After a change of variables this can be more conveniently written as

$$l(t) = \int_0^t \left[D^2 + \left(\frac{dR_m}{dt} \right)^2 \right]^{1/2} dt. \quad (6)$$

The implication of our last assumption is that the wall particle observed passing through $z = 0$, $r = R_m(t)$ at time t must have originated at station $z = Dt - \ell(t)$ on the cylinder wall. If the cylinder is viewed moments later at time,

$$t' \equiv \ell(t)/D > t \quad \text{for } t > 0 ,$$

we see that the expansion has progressed to the situation indicated in Figure 5.b. From this figure we can now deduce the relationship

$$f(t') = f(\ell(t)/D) = \ell(t) - Dt \quad (7)$$

and also, by comparison with Figure 5.a,

$$g(t') = h(0, t). \quad (8)$$

Moreover, the physical significance of t' becomes apparent - it is the time elapsed since the particle observed at station $z = 0$, at time t first commenced its motion. This might be considered as time in a Lagrangian system.

For any choice of t we may compute $P(t)$ and t' from (6) and knowledge of $d(t)$ thereby obtaining from (7) and (8) the position $(f(t'), g(t') + R_m(0))$ which is reached by the cylinder wall particle initially located at $z = 0$, after time t' following passage of the detonation wave. By doing this for sufficiently many choices of t we can reconstruct the entire particle path in the z, r (Eulerian) coordinate system. Henceforth we consider motion in the Lagrangian system, using t' as the independent variable of time; t can be treated as a parameter.

B. Gas Dynamics and Parametric Determination of the Equation of State

Having obtained a complete history of the cylinder wall motion the next step is to reconstruct the Chapman-Jouguet (C-J) isentrope for the explosive detonation products. This is a curve $P = P(V)$, relating pressure to specific volume, which describes the adiabatic expansion from the C-J state (P_{CJ}, V_{CJ})

outwards as far as the experimental data permits. Normally pressure decreases from several hundred to just a few kilobars. Briefly, our procedure for accomplishing this is as follows: 1) differentiation of the particle displacements to obtain velocity and acceleration histories; 2) calculation of pressure as a function of acceleration and cylinder wall strength; 3) determination of density as a function of expansion radius and gas velocity. The apparent radial expansion R_e , measured at discrete choices of time, t , is used parametrically in this process, while R_m , R_a , t' , $f(t')$, $g(t')$, P and V are determined subsequently for each choice of $R_e(t)$, thereby providing one point (P,V) of the isentrope.

The first step, differentiation of the displacement data, requires special numerical treatment and will be discussed separately in the following section.

The pressure of the detonation product gases at any moment follows directly from

$$p = \frac{\text{Force}}{\text{Area}} = \frac{Ma}{2\pi R_a} \quad (9)$$

where M is the mass of the cylinder per unit length and a is the instantaneous acceleration, measured at the center of mass of a wall particle. This is essentially the same as the expression derived by Taylor from a different point of view. The material strength of the cylinder walls has been ignored here but could be taken into account by introducing additional terms; for example,

$$p = \frac{Ma}{2\pi R_a} + \frac{(R_e - R_a)}{2\pi R_a} \left(k_m - k_s \ln \frac{R_e - R_a}{R_1 - R_0} \right) \quad (10)$$

where k_m and k_s denote the engineering and true limit strength of the wall material. These additional terms should really be considered as empirical adjustments since little is known about the strength of materials at strain rates of 10^5 or 10^6 per second. For standard copper cylinders assumed values of k_m or k_s less than 10 kilobars have only minor effects on the pressure/density calculations which we carried out; even at 50 kilobars, the influence of the additional terms in (1) is only seen at the lower end of the $P(V)$ curve ($P \leq 5$ kilobars). Such values are far beyond the measured strength of copper at lesser strain rates so we may assume that material strength is not a critical consideration at this point.

After pressure has been determined as a function of the radial displacement, R_e , the specific volume follows from equations (3) and (11) of Taylor⁴; in our notation these become

$$V = \frac{1}{\rho} = \frac{u}{\rho_0 D} \left(\frac{R}{R_0} \right)^2 \quad (11)$$

where ρ_0 is the explosive loading density, u is the gas velocity given by

$$u = D + \frac{Mw^2}{2\pi\rho_0 R^2 D} - \frac{P}{D\rho_0} \left(\frac{R_a}{R_0} \right)^2 \quad (12)$$

and w is the cylinder wall velocity measured at the central surface

$$w^2(t) = \left[\frac{df(t')}{dt'} \right]^2 + \left[\frac{dg(t')}{dt'} \right]^2 ; \quad (13)$$

the functions $f(t')$ and $g(t')$ are determined as described above.

In carrying out this pressure/density calculation, it is helpful but not necessary to know the detonation pressure in advance either from a separate experiment or analytical predictions, since this provides a starting point for plotting in the $P-V$ plane. Such *a priori* knowledge is usually assumed in the parameter fitting method of EOS formulation, as

discussed earlier. If P_{CJ} is known, then the specific volume at detonation can be determined from (11) and (12) with $w = 0$ and $R_a = R_o$; thus,

$$V_{CJ} = \frac{1}{\rho_o} - \frac{P_{CJ}}{(D\rho_o)^2} \quad (14)$$

A useful quantity which can also be computed after the other variables have been determined is the generalized ratio of specific heats

$$\Gamma = - \left(\frac{\partial \ln P}{\partial \ln V} \right)_S \quad (15)$$

This can be approximated numerically by

$$\Gamma_i = \frac{\ln[P_i/P_{i-1}]}{\ln[V_{i-1}/V_i]} \quad (16)$$

where i is an index of consecutive points on the C-J isentrope. However, this quantity is a bit delicate to compute since it implies third order differentiation of the displacement data and can be expected to produce eccentric results. Nevertheless, with the data modifications outlined in the next section, we have been able to achieve a fairly decent resolution of this function. In contrast, it must be said, an analytic expression of the EOS, with fitting parameters, makes Γ easy to evaluate and smoothly behaved. A good deal of commentary has been directed at interpreting the morphology of Γ in physical terms, but this mostly seems to miss the point, since the appearance and behavior of Γ is as much a consequence of the assumed EOS form as it is of any underlying physical properties. From mathematical analysis we know that two functions can closely agree and still have very dissimilar derivatives. Thus, it is possible to add terms to the EOS formula which cause little effect on subsequent displacement calculations but which strongly affect the appearance of Γ .

C. Numerical Differentiation and Data Adjustment

As anyone who has attempted it knows, differentiation of data obtained from experimental measurements is fraught with difficulties, double differentiation invites disaster and third order derivatives are usually chaotic. This would appear to be true in the present case even though the basic input data appears as smooth as might reasonably be hoped for.

If discrete measurements t_i and $d(t_i)$ are taken from the experimental film record and indexed by i , then let $R_{m,i}$ and t'_i denote the resulting values of R_m and t' determined as explained in Section A. The usual numerical approximations for the radial components of the velocity and acceleration, derived from (4) and (8), are

$$v_i = \frac{R_{m,i+1} - R_{m,i}}{t'_{i+1} - t'_i} \quad (17)$$

$$a_i = 2 \frac{v_i - v_{i-1}}{t'_{i+1} - t'_{i-1}} \quad (18)$$

with analogous formulas for axial motion.

These expressions are exact if the displacement is quadratic in time, which generally is not the case. When these formulas were used in conjunction with (10) and (11) to compute the C-J isentrope for Comp B, grade A, using the test data published in Reference 1 (See Table 1), we obtained the results indicated in Figure 6; the JWL equation of state curve is shown for comparison.

Since this was of little use we had either to abandon or modify the basic procedure. Fortunately, through numerical experimentation, we were able to obtain greatly improved results with only minor corrections to the input data and with a plausible modification of the differentiation formulas.

The first point to be realized is that the published data, shown in columns 1 and 2 of Table 1, lacks precision. The displacement time measurements listed in column 2 are given in units of .01 microseconds. Although this may represent the greatest accuracy achievable with the available equipment it implies round-off errors of $\pm .005$ microseconds, not to mention the inaccuracies in measurement. This may not be significant in itself but does exert considerable

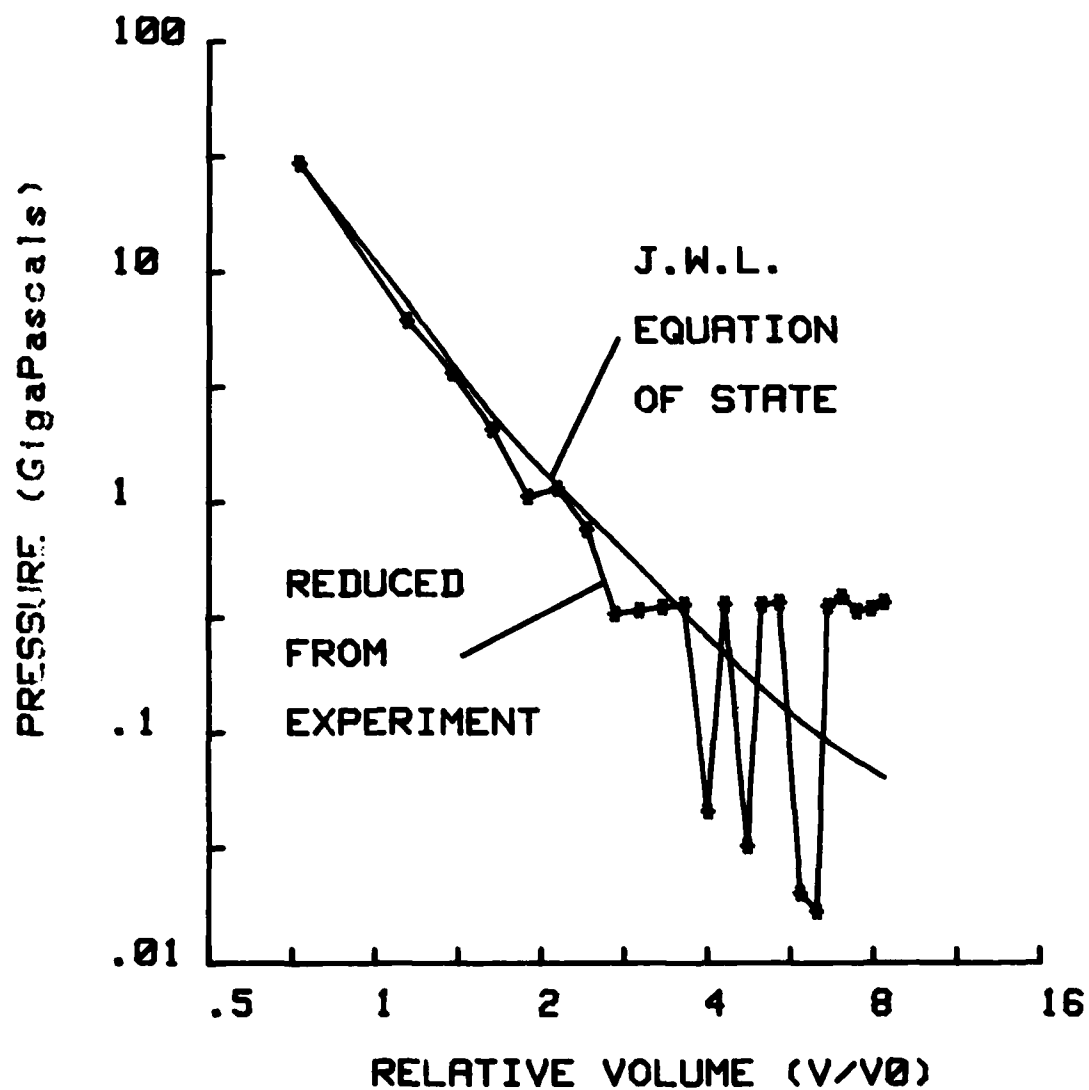


Figure 6. C-J Isentrope Calculated Using Published Data and Algorithm Based on Equations (17) and (18).

influence on the higher order derivatives. It seems reasonable therefore, and not a violation of scientific procedure, to introduce small adjustments ($\leq .005$ microseconds) if this improves the coherence of the subsequent calculations. After all, physical insight tells us that the acceleration of the walls should be a reasonably smooth function, monotonically decreasing with time and displacement. Calculations which show a widely oscillating acceleration must therefore be spurious.

TABLE 1. ORIGINAL AND ADJUSTED DATA FROM LLNL COPPER CYLINDER TEST
REFERENCE 1.

R-RO (mm)	TIME (micro sec)	MODIFIED TIME (micro sec)
2	2.17	2.17
3	3.00	2.997
4	3.77	3.76845
5	4.51	4.5045
6	5.22	5.2160
7	5.91	5.90957
8	6.59	6.5895
9	7.26	7.25875
10	7.92	7.91941
11	8.57	8.57265
12	9.22	9.21929
13	9.86	9.95998
14	10.50	10.49527
15	11.13	11.12567
16	11.75	11.7516
17	12.37	12.3736
18	12.99	12.9921
19	13.60	13.60735
20	14.22	14.21958
21	14.83	14.8290
22	15.43	15.43575
23	16.04	16.04

TABLE 2. CALCULATED VALUES FOR ACCELERATION
AND Γ DURING CYLINDER EXPANSION

Expm't Displcm't	(1) Original Data; formulas (17)&(18)		(2) Modified Data; formulas (17)&(18)		(3) Modified Data; formulas (21)&(22)	
	R-Ro	Acc Γ	Acc Γ	Acc Γ	Acc Γ	Acc Γ
0.0		.000 2.49	.000 2.49	.000 2.49	.000 2.49	.000 2.49
1.0		.773 3.07	.773 3.07	.720 2.41	.720 2.41	.720 2.41
2.0		.252 3.94	.252 3.94	.261 3.60	.261 3.60	.261 3.60
3.0		.141 3.51	.146 3.35	.145 3.55	.145 3.55	.145 3.55
4.0		.109 2.19	.100 2.89	.099 2.93	.099 2.93	.099 2.93
5.0		.060 4.48	.073 2.71	.072 2.78	.072 2.78	.072 2.78
6.0		.070 -.67	.056 2.66	.054 2.71	.054 2.71	.054 2.71
7.0		.049 3.32	.043 2.66	.042 2.70	.042 2.70	.042 2.70
8.0		.023 7.10	.034 2.62	.033 2.65	.033 2.65	.033 2.65
9.0		.025 -.36	.027 2.62	.027 2.63	.027 2.63	.027 2.63
10.0		.027 -.31	.023 2.55	.022 2.55	.022 2.55	.022 2.55
11.0		.029 -.31	.020 1.79	.020 1.73	.020 1.73	.020 1.73
12.0		.005 20.22	.018 1.48	.018 1.47	.018 1.47	.018 1.47
13.0		.032 -27.81	.017 1.38	.017 1.38	.017 1.38	.017 1.38
14.0		.004 24.68	.016 1.36	.016 1.36	.016 1.36	.016 1.36
15.0		.034 -35.93	.015 1.48	.015 1.49	.015 1.49	.015 1.49
16.0		.036 -.30	.014 1.40	.014 1.40	.014 1.40	.014 1.40
17.0		.003 31.42	.012 2.19	.012 2.22	.012 2.22	.012 2.22
18.0		.003 2.18	.011 2.06	.011 2.03	.011 2.03	.011 2.03
19.0		.039 -63.55	.011 1.33	.010 1.28	.010 1.28	.010 1.28
20.0		.043 -1.44	.010 1.38	.010 1.38	.010 1.38	.010 1.38
21.0		.039 2.23	.010 1.04	.010 1.03	.010 1.03	.010 1.03
22.0		.041 -.46	.009 .99	.009 1.03	.009 1.03	.009 1.03
23.0		-- --	-- --	.009 .93	.009 .93	.009 .93

These calculations have been done three different ways as noted: Either the original or modified displacement data from Table 1 was used, and either formulas (17) and (18) or formulas (21) and (22) were used for the acceleration calculation.

After some trial and error adjustments of the original data, we obtained the modified data indicated in column 3 of Table 1 as a more precise record of the actual cylinder wall displacement. The adjusted and unadjusted values of acceleration versus time are given in Table 2. It is clear here that the modified values exhibit a much more reasonable behavior than the original ones and are probably a more accurate description of the actual physical event. The resulting P-V curve is shown in Figure 7. We wish to stress that the modified data in column 3 does not contradict the original data but merely enhances it - makes it more precise. Except in three cases, the adjustment is less than the round-off error of .005 microseconds and the maximum adjustment is only .007 microseconds. Essentially, we have used the data to correct itself in a manner which leads to monotonically decreasing second derivatives.

Since a fair amount of discrepancy still remained between our calculated values and the JWL curve, as shown in Figure 7, we considered whether additional changes in the procedure might yield improved results. One possibility was to change the velocity and acceleration formulas (17) and (18); as mentioned, these are exact for displacements which are quadratic in time, implying a constant acceleration over the time interval. In actuality, the acceleration is monotonically decreasing with respect to time and displacement, so we might suppose that over a time interval $t_0 \leq t \leq t_1$, the displacement is better described in the form

$$R_m(t) = d_0 + v_0(t - t_0) + c(t - t_0)^b \quad (19)$$

where d_0 and v_0 are the displacement and velocity at $t = t_0$. Positive, strictly decreasing acceleration requires that

$$1 < b < 2 \quad \text{and} \quad c > 0$$

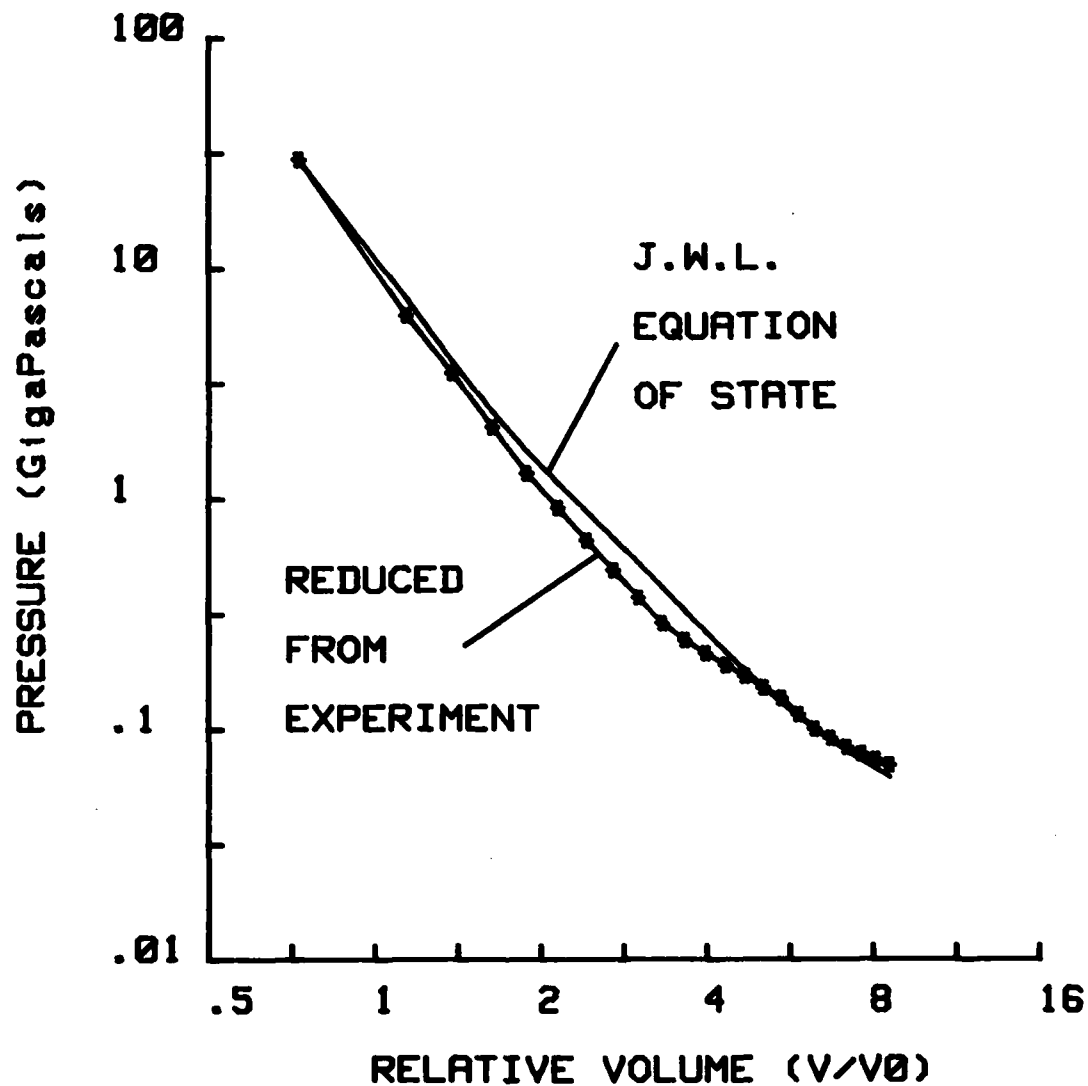


Figure 7. C-J Isentrope Calculated Using Adjusted Data and Algorithm Based on Equations (17) and (18).

and in fact $b = 1.5$ seems to produce the most satisfactory results in practice. If d_0 and v_0 are known and if

$$R_m(t_1) = d_1$$

is specified then

$$c = (d_1 - d_0 - v_0 \Delta t) / (\Delta t)^b \quad (20)$$

where $\Delta t = t_1 - t_0$. The acceleration at mid-interval, $t = (t_0 + t_1)/2$, is

$$a = 2^{2-b}(b-1)(d_1 - d_0 - v_0 \Delta t) / \Delta t^2 \quad (21)$$

and the velocity at $t = t_1$ can then be evaluated as

$$v_1 = v_0 + a \Delta t. \quad (22)$$

At the start of the first time interval, the displacement and velocity are known to be zero; the average acceleration a and the velocity v_1 can thus be determined in terms of d_1 . Formulas (21) and (22) can then be applied recursively to determine the average acceleration over each subsequent time interval.

When this revised procedure is applied to the modified displacement data from Table 1, we obtain the acceleration history indicated in Table 2 and the EOS curve shown in Figure 8. This appears to be a significant

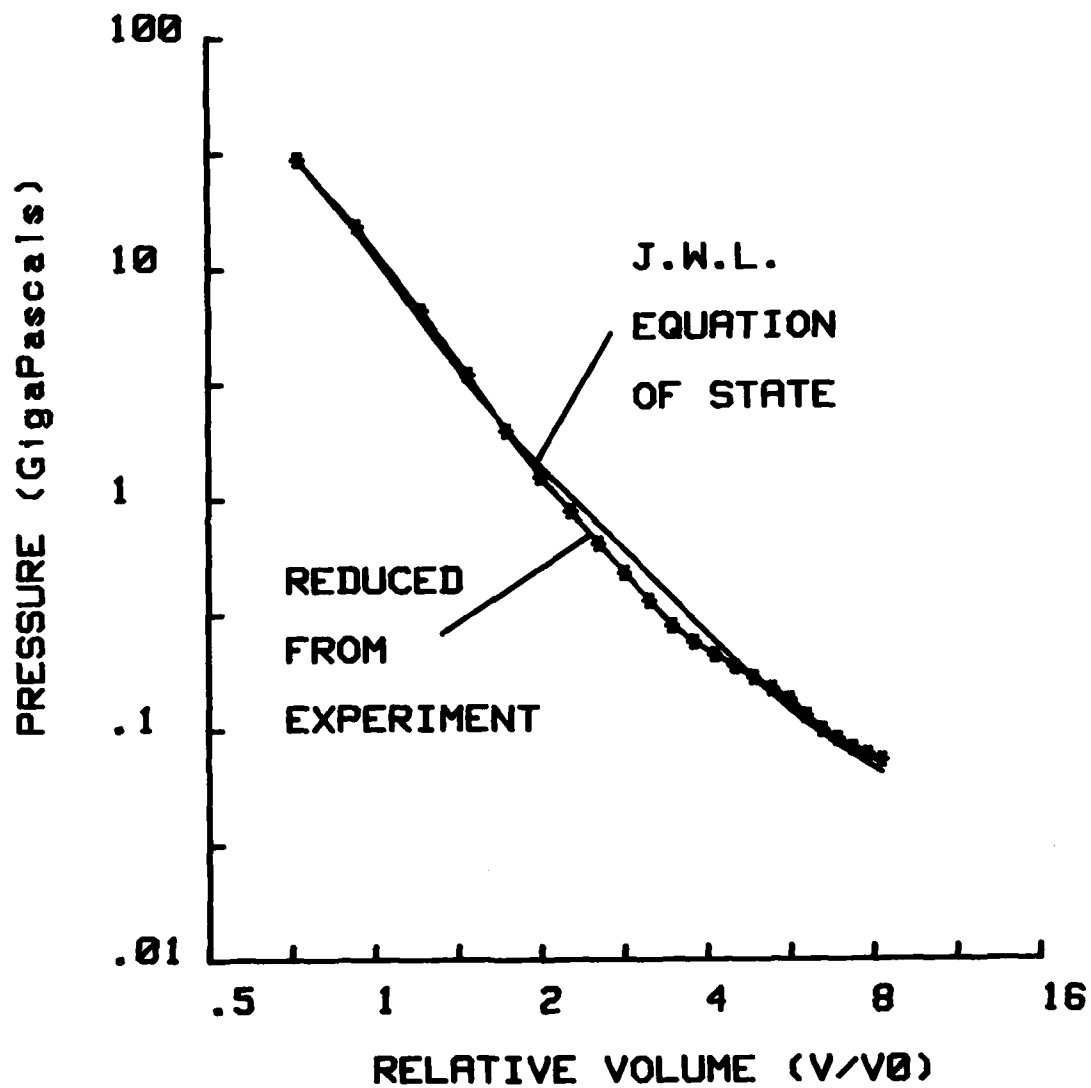


Figure 8. C-J Isentrope Calculated Using Adjusted Data and Algorithm Based on Equations (21) and (22).

improvement over the more straightforward approach using equation (17) and (18) (assuming the JWL equation of state for Comp B, grade A, is valid). We believe that it is noteworthy that no further modification of the displacement data was necessitated by this fundamental change in algorithm and take this as additional evidence of the suitability of the modified data.

IV. DISCUSSION AND CONCLUSIONS

The procedure described in this report provides a means for extracting equation of state data directly from cylinder expansion test measurements and should provide a useful tool to researchers in explosion dynamics. Its principal advantage is in eliminating the dependence on large scale continuum mechanics code calculations and contrived mathematical expressions for the equation of state. It must be said however, that this procedure still is a long way from being confidently established and additional work is required to accomplish this.

Our objective in the present report has been to derive and elucidate the mathematical formulations and methodology. Thus we have only considered the single case of Comp B, grade A, in a standard copper cylinder. To complete the task a thorough comparison of this procedure against known results needs to be carried out. In particular:

- 1) The method should be applied to the cylinder test data for a variety of explosives and results compared with conventional EOS data.

- 2) The EOS subroutines of continuum mechanics codes should be modified to accept the tabular data produced by the present method, in place of the usual analytic expression; calculations could then be performed and compared with the previous results and with experiments.
- 3) Sensitivity of the procedure to variation in experimental parameters (cylinder material and dimensions, explosive loading density) should be investigated.
- 4) A method for modifying the basic input data should be systematized, if possible, so that experimenters do not have to waste time on trial-and-error data adjustments. In this regard it may be possible to predetermine the properties of Γ (see equation (15)); that is, to carry out data adjustments subject to restrictions on the range, fluctuation or asymptotic behavior of Γ .

From the mathematical point of view we believe that there is an interesting and practical question, worthy of a separate investigation, which underlies the development of the present algorithm and relates to the general problem of data differentiation. We might state this as follows:

Let the values of a function be given at discrete choices of its argument, let $E > 0$ be a given error bound and $N > 0$ be an integer. How, and when, can the given data be approximated by a fitting function that differs from the prescribed values by at most E and which possesses a non-negative derivative of order N in the region of interest?

The data adjustment procedure of Section III.C amounts to an empirical solution of this problem but the question is certainly of broader interest.

ACKNOWLEDGEMENT

The author would like to express his appreciation to Mr. Walter Smothers of Range 7A for conducting the experimental work and sharing his experience, to Mr. John Kineke for arranging and coordinating the tests and Drs. James Dehn and Anthony Finnerty for first introducing me to the cylinder expansion test procedure.

APPENDIX A
TABULATION OF COMPUTER PROGRAM "CEDAR"

TABULATION OF COMPUTER PROGRAM "CEDAR"

```

1      ! RE-STORE "CEDAR:F"
2 PRINT "*****"
3 PRINT "***** C E D A R *****"
4 PRINT "***** CYLINDER EXPANSION DATA REDUCTION PROGRAM *****"
5 PRINT "*****"
6 PRINT
7 PRINTER IS 16
8 ! PRINTER IS 0
10 REM Unit of length: millimeters
20 REM Unit of time: microsecond
30 REM Unit of mass: gram
40 REM Unit of velocity: mm/microsec (=Km/sec)
50 REM Unit of acceleration: mm/microsec2
60 REM Unit of density: gm/mm3
70 REM Unit of pressure: terapascal (= 10 megabars)
80 REM Input pressure in gigapascals (= 10 kilobars)
90 REM Output pressure displayed in gigapascals
100 DIM Disp(100),Ta(100),Ca(100),Rm(100),Tp(100),Gamma(100),Ps(100)
110 DIM T(100),Ra(100),Z(100),V(100),P(100),Rho(100)
120 DIM Vr(100),Vz(100),Vgas(100),Vol(100)
130 DIM Re(100),Ve(100),Ae(100),Acc(100)
200 D=7.98          ! DETONATION VELOCITY
210 Dc=.001717      ! EXPLOSIVE LOADING DENSITY
220 Dm=.008940      ! CYLINDER WALL DENSITY
240 Km=1            ! WALL STRENGTH PARAMETER (EQUATION 10)
250 Ks=1            ! WALL STRENGTH PARAMETER (EQUATION 10)
260 REM **** ENTER Km,Ks IN GIGAPASCALS ****
270 Psc=.001
280 Km=Km*Psc
290 Ks=Ks*Psc
300 REM **** Km,Ks NOW IN COMPATIBLE UNITS (terapascals) ****
310 R0=12.7
320 R1=R0+2.606
330 N=26
340 P0=29.5          ! EXPLOSIVE DETONATION PRESSURE
350 REM **** ENTER P0 IN GIGAPASCALS ****
360 P0=P0*Psc
370 REM **** P0 NOW IN COMPATIBLE UNITS (terapascals) ****
380 AO=PI*(R12-R02)    ! CROSSECTIONAL AREA OF CYLINDER WALL
390 Rm0=SQR((R02+R12)/2) ! INITIAL RADIAL COORDINATE OF CENTRAL SURFACE
400 M=Dm*AO           ! CYLINDER MASS PER UNIT LENGTH
410 C=Dc*PI*R02       ! EXPLOSIVE MASS PER UNIT LENGTH

```

Note: The symbol "e" in this tabulation denotes exponentiation.

```

500 REM ***** READ IN TEST DATA *****
510 FOR I=1 TO N
520 READ Disp(I),Ta(I)
530 Re(I)=R1+Disp(I)
540 NEXT I
550 DATA 0.00,-2.00,0.00,-1.00,0.00,-.140,1.00,1.212
551 DATA 2.,2.17
552 DATA 3.,3.00
553 DATA 4.,3.77
554 DATA 5.,4.51
555 DATA 6.,5.22
556 DATA 7.,5.91
557 DATA 8.,6.59
558 DATA 9.,7.26
559 DATA 10.,7.92
560 DATA 11.,8.57
561 DATA 12.,9.22
562 DATA 13.,9.86
563 DATA 14.,10.50
564 DATA 15.,11.13
565 DATA 16.,11.75
566 DATA 17.,12.37
567 DATA 18.,12.99
568 DATA 19.,13.60
569 DATA 20.,14.22
570 DATA 21.,14.83
571 DATA 22.,15.43
572 DATA 23.,16.04
600 REM ***** INPUT DATA SMOOTHING FACTORS *****
601 ! GOTO 700 ! SKIP TO 700 TO OMIT DATA SMOOTHING
610 FOR I=1 TO N
620 READ Ca(I)
630 Ta(I)=Ta(I)+Ca(I)
640 NEXT I
650 DATA 0,0,0,0,-0.0040
651 DATA -.00300,-.00155,-.00550,-.00400,-.00043
652 DATA -.00050,-.00125,-.00059,+.00265,-.00071
653 DATA -.00002,-.00473,-.00433,+.00160,+.00360
654 DATA +.00210,+.00735,-.00042,-.00105,+.00560
655 DATA -.00035
700 PRINT "***** INPUT CYLINDER EXPANSION DATA *****"
710 PRINT " I R-RO T Va Aa"
720 Ve(0)=0
730 Ae(1)=0
740 Rm(1)=Rm0
750 Ra(1)=R0
800 REM ***** CALCULATE APPARENT VELOCITY AND ACCELERATION *****
810 FOR I=1 TO N
820 Ve(I)=(Re(I+1)-Re(I))/(Ta(I+1)-Ta(I)) ! EXTERIOR WALL VELOCITY
830 Ae(I)=2*(Ve(I)-Ve(I-1))/(Ta(I+1)-Ta(I-1)) ! EXTERIOR WALL ACCELERATION
840 Rm(I)=SQR(Re(I)*2-A0/(2*PI)) ! CENTRAL SURFACE DISPLACEMENT
850 Ra(I)=SQR(Re(I)*2-A0/PI) ! INTERIOR WALL DISPLACEMENT
860 PRINT USING 861;I,Disp(I),Ta(I),Ve(I),Ae(I)
861 IMAGE DDD,4(DDD.DDDD)
870 NEXT I
880 PRINT

```

```

890 PRINT " I = INDEX OF DATA POINT READING FROM FILM RECORD"
900 PRINT "R-RO = MEASURED EXTERIOR WALL DISPLACEMENT (MM)"
910 PRINT " T = MEASURED EXPANSION TIME (MICROSECONDS)"
911 PRINT " Va = APPARENT EXTERIOR WALL VELOCITY (MM/MICROSEC)"
912 PRINT " Aa = APPARENT EXTERIOR WALL ACCELERATION (MM/MICROSEC^2)"
913 PRINT
1000 REM ***** CALCULATE LAGRANGIAN TIME SCALE Tp(I) *****
1001 PRINT "***** CALCULATED DATA *****"
1010 Tp(1)=0
1020 Z(1)=0
1030 Vr(1)=Vz(1)=V(1)=0
1040 P(1)=P0
1050 Vgas(1)=0
1060 Vol(1)=1/Dc-P0/(D*Dc)^2
1070 Rho(1)=1/Vol(1)
1080 Gamma(1)=Dc*D^2/P0-1
1090 PRINT " I Rm-Rm0 Z Tp Vr Vz Ar Ac Rho P
Gamma"
1091 IMAGE DDD,3(DDD.DDD),2X,3(DDD.DDD),DDD.DDD,D.DDDDD,DDDD.DD,DDD.DD
1092 GOTO 1200 ! SKIP TO 1200 TO USE IMPROVED ACCELERATION ALGORITHM
1100 REM ***** CALCULATION BASED ON STANDARD ACCELERATION ALGORITHM *****
1110 Tdel=SQR((Ta(2)-Ta(1))^2+((Rm(2)-Rm(1))/D)^2) ! LAGRANGIAN TIME INCREMENT
1112 Tp(2)=Tp(1)+Tdel ! TIME ELAPSED IN MOTION
1114 Z(2)=D*(Tp(2)-Ta(2)+Ta(1)) ! LAGRANGIAN AXIAL DISPLMNT
1120 FOR I=2 TO N-1
1122 Tdel=SQR((Ta(I+1)-Ta(I))^2+((Rm(I+1)-Rm(I))/D)^2) ! LAGRANGIAN TIME INCR.
1124 Tp(I+1)=Tp(I)+Tdel ! TIME ELAPSED IN MOTION
1126 Z(I+1)=D*(Tp(I+1)-Ta(I+1)+Ta(1)) ! LAGRANGIAN AXIAL DISPLACEMENT
1130 Vr(I)=(Rm(I+1)-Rm(I))/Tdel ! LAGRANGIAN RADIAL VELOCITY
1132 Vz(I)=(Z(I+1)-Z(I))/Tdel ! LAGRANGIAN RADIAL VELOCITY
1134 V(I)=SQR(Vr(I)^2+Vz(I)^2) ! VELOCITY ALONG PARTICLE PATH
1140 Ar=2*(Vr(I)-Vr(I-1))/(Tp(I+1)-Tp(I-1)) ! LAGRANGIAN RADIAL ACCELERATION
1142 Az=2*(Vz(I)-Vz(I-1))/(Tp(I+1)-Tp(I-1)) ! LAGRANGIAN AXIAL ACCELERATION
1144 Acc(I)=SQR(Ar^2+Az^2) ! ACCELERATION ALONG PARTICLE PATH
1146 IF Ra(I)<=Ra(I-1) THEN 1180
1150 P(I)=M*Acc(I)/(2*PI*Ra(I))
1152 P(I)=P(I)*(1+(V(I)/D)^2)^(-1.5) ! EXPLOSIVE PRODUCTS PRESSURE
1154 Rdel=Re(I)-Ra(I)
1156 P(I)=P(I)-Rdel*(Km-Ks*LOG(Rdel/(R1-RO)))/(2*PI*Ra(I))
1160 Vgas(I)=D*M*V(I)^2/(2*PI*D*Dc*RO^2)-P(I)*(Ra(I)/RO)^2/(D*Dc)
1162 ! Vol(I)=1+.5*(M/C)*(V(I)/D)^2-P(I)/(Dc*D^2)*(Ra(I)/RO)^2
1164 ! Vol(I)=Vol(I)/Dc*(Ra(I)/RO)^2
1166 Vol(I)=Vgas(I)*Ra(I)^2/(Dc*D*RO^2) ! SPECIFIC VOLUME
1168 Rho(I)=1/Vol(I) ! GAS DENSITY
1170 IF P(I)>0 THEN 1174
1172 P(I)=.000001
1174 Gamma(I)=LOG(P(I)/P(I-1))/LOG(Vol(I-1)/Vol(I)) ! GENRL. RAT.OF SPEC.HEAT
1176 GOTO 1190
1180 P(I)=P(I-1)
1182 Vgas(I)=Vgas(I-1)
1184 Vol(I)=Vol(I-1)
1186 Rho(I)=1/Vol(I)
1188 Gamma(I)=Gamma(I-1)
1190 PRINT USING 1091;I,Rm(I)-Rm0,Z(I),Tp(I),Vr(I),Vz(I),Ar,Az,Rho(I),P(I)/Psc,
Gamma(I)
1192 NEXT I
1194 GOTO 1500

```

```

1200 REM ***** CALCULATION BASED ON IMPROVED ACCELERATION ALGORITHM *****
1205 FOR I=2 TO N
1210 Tdel=SQR((Ta(I)-Ta(I-1))2+((Rm(I)-Rm(I-1))/D)2) ! LAGRANGIAN TIME INCR.
1220 Tp(I)=Tp(I-1)+Tdel ! TIME ELAPSED SINCE MOTION COMMENCED
1230 Z(I)=D*(Tp(I)-Ta(I)+Ta(1)) ! LAGRANGIAN RADIAL ACCELERATION
1240 Ar=Rm(I)-Rm(I-1)-Vr(I-1)*Tdel
1242 Ar=1.06*Ar/Tdel2 ! LAGRANGIAN RADIAL ACCELERATION
1250 Az=Z(I)-Z(I-1)-Vz(I-1)*Tdel
1252 Az=1.06*Az/Tdel2 ! LAGRANGIAN AXIAL ACCELERATION
1260 Acc(I)=SQR(Ar2+Az2) ! ACCELERATION ALONG PARTICLE PATH
1270 Vr(I)=Vr(I-1)+Ar*Tdel ! RADIAL VELOCITY, INDEX I
1272 Vrm=Vr(I-1)+.5*Ar*Tdel ! RADIAL VELOCITY AT MIDPOINT
1280 Vz(I)=Vz(I-1)+Az*Tdel ! AXIAL VELOCITY, INDEX I
1282 Vzm=Vz(I-1)+.5*Az*Tdel ! AXIAL VELOCITY AT MIDPOINT
1290 V(I)=SQR(Vrm2+Vzm2) ! AVERAGE VELOCITY ALONG PATH
1300 IF Ra(I)<=Ra(I-1) THEN 1400
1310 Raa=.5*(Ra(I-1)+Ra(I)) ! AVERAGE INNER WALL DISPLACEMENT
1312 Rea=.5*(Re(I-1)+Re(I)) ! AVERAGE OUTER WALL DISPLACEMENT
1314 Rdel=Rea-Raa ! WALL THICKNESS
1320 P(I)=M*Acc(I)/(2*PI*Raa) ! DETONATION PRODUCTS PRESSURE
1322 P(I)=P(I)*(1+(V(I)/D)2)(-1.5)
1324 P(I)=P(I)-Rdel*(Km-Ks*LOG(Rdel/(R1-RO)))/(2*PI*Raa)
1330 Vgas(I)=D*M*V(I)2/(2*PI*D*Dc*RO2)-P(I)*(Raa/RO)2/(D*Dc)
1340 ! Vol(I)=1+.5*(M/C)*(V(I)/D)2-P(I)/(Dc*D2)*(Raa/RO)2
1342 ! Vol(I)=Vol(I)/Dc*(Raa/RO)2
1346 Vol(I)=Vgas(I)*Raa2/(Dc*D*RO2) ! GAS SPECIFIC VOLUME
1350 Rho(I)=1/Vol(I) ! GAS DENSITY
1360 IF P(I)>0 THEN 1380
1370 P(I)=.000001
1380 Gamma(I)=LOG(P(I)/P(I-1))/LOG(Vol(I-1)/Vol(I)) ! GENRL.RATIO OF SPEC.HEAT
1390 GOTO 1450
1400 P(I)=P(I-1)
1410 Vgas(I)=Vgas(I-1)
1420 Vol(I)=Vol(I-1)
1430 Rho(I)=1/Vol(I)
1440 Gamma(I)=Gamma(I-1)
1450 PRINT USING 1091;I,Rm(I)-Rm0,Z(I),Tp(I),Vr(I),Vz(I),Ar,Az,Rho(I),P(I)/Psc,
Gamma(I)
1460 NEXT I
1500 PRINT
1510 PRINT "Rm-Rm0 = RADIAL DISPLACEMENT OF WALL PARTICLE CENTER OF MASS (MM)"
1520 PRINT " Z = AXIAL DISPLACEMENT OF WALL PARTICLE CENTER OF MASS (MM)"
1530 PRINT " Tp = ELAPSED TIME SINCE START OF PARTICLE MOTION (MICROSEC)"
1540 PRINT " Vr = LAGRANGIAN RADIAL VELOCITY COMPONENT (MM/MICROSEC)"
1550 PRINT " Vz = LAGRANGIAN AXIAL VELOCITY COMPONENT (MM/MICROSEC)"
1560 PRINT " Ar = LAGRANGIAN RADIAL ACCELERATION COMPONENT (MM/MICROSEC2)"
1570 PRINT " Az = LAGRANGIAN AXIAL ACCELERATION COMPONENT (MM/MICROSEC2)"
1580 PRINT " Rho = DETONATION PRODUCTS GAS DENSITY (GRAMS/MM3)"
1590 PRINT " P = DETONATION PRODUCTS GAS PRESSURE (GIGAPASCALS)"
1600 PRINT " Gamma = GENERALIZED RATIO OF SPECIFIC HEATS"
1610 PRINT
1620 PRINT "END OF COMPUTATION"
3000 REM ***** PLOT PRESSURE VS SPECIFIC VOLUME *****
3010 GRAPHICS
3011 ! GOTO 3020 ! SKIP TO 3020 TO USE 9872A PLOTTER
3012 PLOTTER IS 13,"GRAPHICS"

```

```

3013 LIMIT 0,180,0,131
3014 GOTO 3030
3020 PLOTTER IS 7,5,"9872A"
3021 LIMIT 0,200,25,250
3030 LOCATE 25,95,15,80
3040 CSIZE 3
3050 SCALE LOG(.5),LOG(32),LOG(.01),LOG(100)
3060 CLIP LOG(.5),LOG(16),LOG(.01),LOG(100)
3070 AXES LOG(SQR(2)),LOG(10000)/8,LOG(.5),LOG(.01)
3071 OUTPUT 705;"SM"
3072 LINE TYPE 1
3073 FOR I=2 TO N-1
3074 PLOT LOG(Dc*Vol(I)),LOG(P(I)/Psc)    ! PLOT DATA POINT ALONG C-J ISENTROPE
3075 NEXT I
3080 OUTPUT 705;"SM"
3084 PENUP
3085 LINE TYPE 1
3090 REM PLOT C-J ISENTROPE FROM JWEL EQUATION OF STATE
3091 A=5.24229
3092 B=.076783
3093 R1=4.2
3094 R2=1.1
3095 Omega=.34
3096 FOR I=1 TO N-1
3097 Ps(I)=A*EXP(-R1*Vol(I)*Dc)+B*EXP(-R2*Vol(I)*Dc)+.0108*(Vol(I)*Dc)0(-(1+Omega))
3098 PLOT LOG(Vol(I)*Dc),LOG(Ps(I)*100)
3099 NEXT I
3100 FOR I=0 TO 5
3110 MOVE LOG(20(I-1))+LOG(.9),LOG(.006)
3120 LABEL 20(I-1)
3130 NEXT I
3140 FOR I=0 TO 4
3150 MOVE LOG(.3),(I-2)*LOG(10)
3160 LABEL 100(I-2)
3170 NEXT I
3180 MOVE LOG(1),LOG(.003)
3190 LABEL "RELATIVE VOLUME (V/V0)"
3200 LDIR PI/2
3210 MOVE LOG(.25),LOG(.1)
3220 LABEL "PRESSURE (GigaPascals)"
3221 LDIR 0
3230 MOVE LOG(3),LOG(10)
3231 LABEL "J.W.L."
3232 MOVE LOG(3),LOG(5)
3233 LABEL "EQUATION"
3234 MOVE LOG(3),LOG(2.5)
3235 LABEL "OF STATE"
3250 MOVE LOG(.7),LOG(.4)
3251 LABEL "REDUCED"
3252 MOVE LOG(.7),LOG(.2)
3253 LABEL "FROM"
3254 MOVE LOG(.7),LOG(.1)
3255 LABEL "EXPERIMENT"
3500 LDIR 0
3510 PEN 0
3520 STOP

```

APPENDIX B
OUTPUT FROM COMPUTER PROGRAM "CEDAR"

OUTPUT FROM COMPUTER PROGRAM "CEDAR"

 ***** C E D A R *****
 ***** CYLINDER EXPANSION DATA REDUCTION PROGRAM *****

***** INPUT CYLINDER EXPANSION DATA *****

I	R-RO	T	Va	Aa
1	0.0000	-2.0000	0.0000	0.0000
2	0.0000	-1.0000	0.0000	0.0000
3	0.0000	-.1400	.7396	.6688
4	1.0000	1.2120	1.0482	.2676
5	2.0000	2.1660	1.2034	.1738
6	3.0000	2.9970	1.2963	.1159
7	4.0000	3.7685	1.3586	.0827
8	5.0000	4.5045	1.4055	.0648
9	6.0000	5.2160	1.4418	.0517
10	7.0000	5.9096	1.4707	.0421
11	8.0000	6.5895	1.4942	.0348
12	9.0000	7.2588	1.5136	.0292
13	10.0000	7.9194	1.5308	.0262
14	11.0000	8.5727	1.5465	.0240
15	12.0000	9.2193	1.5608	.0223
16	13.0000	9.8600	1.5741	.0208
17	14.0000	10.4953	1.5863	.0193
18	15.0000	11.1257	1.5976	.0180
19	16.0000	11.7516	1.6077	.0162
20	17.0000	12.3736	1.6168	.0147
21	18.0000	12.9921	1.6254	.0138
22	19.0000	13.6074	1.6334	.0131
23	20.0000	14.2196	1.6410	.0126
24	21.0000	14.8290	1.6484	.0121
25	22.0000	15.4356	1.6555	.0117
26	23.0000	16.0397	2.3882	-.0949

I = INDEX OF DATA POINT READING FROM FILM RECORD
 R-RO = MEASURED EXTERIOR WALL DISPLACEMENT (MM)
 T = MEASURED EXPANSION TIME (MICROSECONDS)
 Va = APPARENT EXTERIOR WALL VELOCITY (MM/MICROSEC)
 Aa = APPARENT EXTERIOR WALL ACCELERATION (MM/MICROSEC²)

***** CALCULATED DATA *****

I	Rm-Rm0	Z	Tp	Vr	Vz	Ar	Ac	Rho	P	Gamma
2	0.000	0.000	1.000	0.000	0.000	0.000	0.000	.00235	29.50	2.71
3	0.000	0.000	1.860	0.000	0.000	0.000	0.000	.00235	29.50	2.71
4	1.082	.054	3.219	.844	.042	.621	.031	.00184	15.17	2.72
5	2.154	.129	4.182	1.128	.080	.295	.039	.00141	6.53	3.13
6	3.217	.214	5.024	1.271	.102	.170	.026	.00116	3.44	3.32
7	4.273	.304	5.807	1.354	.116	.105	.018	.00099	1.98	3.43
8	5.323	.397	6.554	1.407	.125	.072	.013	.00086	1.25	3.26
9	6.368	.492	7.278	1.447	.132	.054	.010	.00075	.89	2.70
10	7.409	.589	7.983	1.476	.138	.042	.008	.00067	.64	2.75
11	8.446	.688	8.676	1.499	.142	.033	.006	.00060	.48	2.71
12	9.480	.787	9.357	1.518	.146	.027	.005	.00054	.36	2.70
13	10.511	.887	10.031	1.532	.149	.022	.004	.00049	.28	2.64
14	11.540	.987	10.696	1.546	.151	.020	.004	.00045	.24	1.79
15	12.566	1.088	11.356	1.557	.153	.018	.004	.00041	.21	1.52
16	13.590	1.190	12.009	1.568	.156	.017	.003	.00038	.19	1.42
17	14.613	1.292	12.657	1.579	.158	.016	.003	.00035	.17	1.41
18	15.634	1.395	13.300	1.588	.160	.015	.003	.00033	.15	1.55
19	16.654	1.498	13.939	1.597	.161	.014	.003	.00030	.13	1.45
20	17.673	1.601	14.574	1.605	.163	.012	.002	.00028	.11	2.35
21	18.690	1.705	15.206	1.611	.164	.011	.002	.00026	.10	2.15
22	19.706	1.809	15.834	1.618	.166	.010	.002	.00025	.09	1.34
23	20.722	1.913	16.459	1.624	.167	.010	.002	.00023	.08	1.45
24	21.736	2.018	17.082	1.630	.168	.010	.002	.00022	.08	1.06
25	22.750	2.123	17.702	1.636	.169	.009	.002	.00021	.07	1.03
26	23.763	2.228	18.319	1.642	.171	.009	.002	.00020	.07	.95

Rm-Rm0 = RADIAL DISPLACEMENT OF WALL PARTICLE CENTER OF MASS (MM)

Z = AXIAL DISPLACEMENT OF WALL PARTICLE CENTER OF MASS (MM)

Tp = ELAPSED TIME SINCE START OF PARTICLE MOTION (MICROSEC)

Vr = LAGRANGIAN RADIAL VELOCITY COMPONENT (MM/MICROSEC)

Vz = LAGRANGIAN AXIAL VELOCITY COMPONENT (MM/MICROSEC)

Ar = LAGRANGIAN RADIAL ACCELERATION COMPONENT (MM/MICROSEC²)

Az = LAGRANGIAN AXIAL ACCELERATION COMPONENT (MM/MICROSEC²)

Rho = DETONATION PRODUCTS GAS DENSITY (GRAMS/MM³)

P = DETONATION PRODUCTS GAS PRESSURE (GIGAPASCALS)

Gamma = GENERALIZED RATIO OF SPECIFIC HEATS

END OF COMPUTATION

Note: The symbol "e" denotes exponentiation.

LIST OF SYMBOLS

Symbol	Meaning	Page
A_0	Cross-sectional area of cylinder wall	13
D	Detonation velocity of explosive	13
$f(t)$	Axial displacement	17
$g(t)$	Radial displacement	17
$h(z,t)$	Central surface displacement observed at station z , time t	13
$\ell(t)$	Arclength along path of particle motion	17
M	Mass of cylinder per unit length	19
P	Pressure of detonation product gases	20
r	Eulerian radial coordinate	13
R_0	Initial inner radius of cylinder wall	13
R_1	Initial exterior radius of cylinder wall	13
$R_a(t)$	Inner radius of cylinder wall at time t	13
$R_e(t)$	Exterior radius of cylinder wall at time t	13
$R_m(t)$	Radial displacement of central surface at time t	13
t	Time as measured from film record	12
t'	Elapsed time since particle observed at time t , station z , began its motion	18
u	velocity of detonation product	20
v	Radial velocity component of cylinder wall motion	22
V	Specific volume of detonation product gases	19
w	Cylinder wall velocity	20
z	Eulerian axial coordinate	13
Γ	Generalized ratio of specific heats	21
ρ	Density of detonation product gases	20

Subscript	Significance	Page
o	Initial values before detonation	13
a	Interior of cylinder wall	13
CJ	Chapman-Jouguet values, immediately following detonation	18
e	Exterior of cylinder wall	13
i	Index of discrete data points, from film reader	13
m	Central surface of cylinder	13
S	Values along C-J isentrope	21

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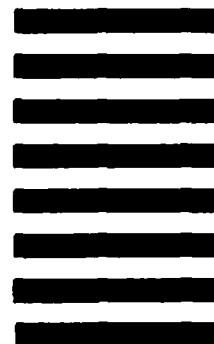


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